Note Title 9/26/2006

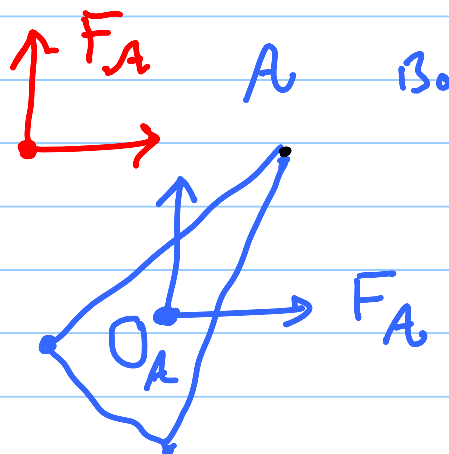
Lecture 7

Specific cases for computing C_{obs}:

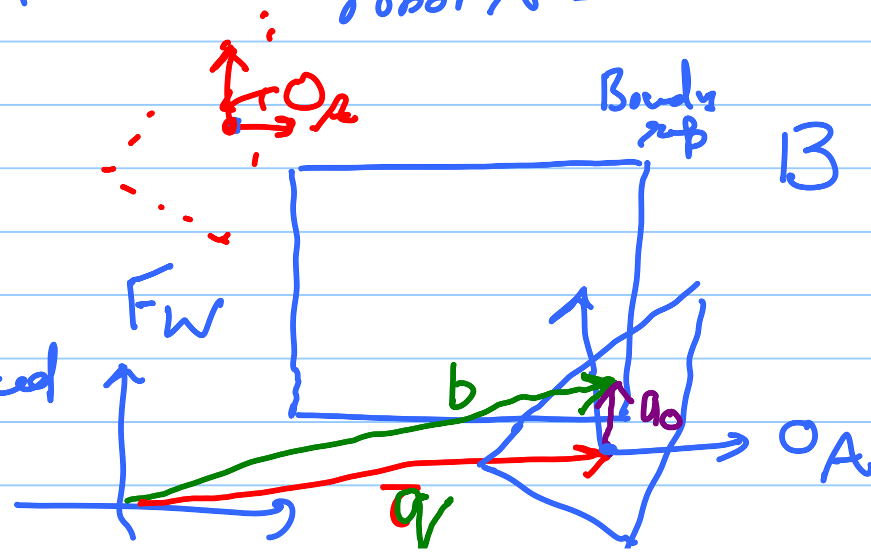
1) convex polygons: robot, obs,

$B_i :=$ convex poly

robot A = " " , pure trans.

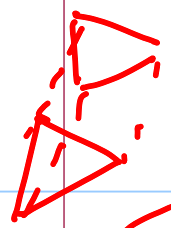


$\underline{\Theta}$ is fixed



$b = q + a_0$

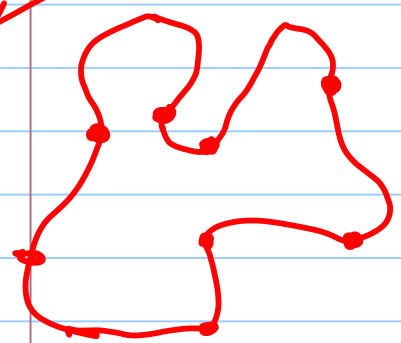
$\Rightarrow q = b - a_0$

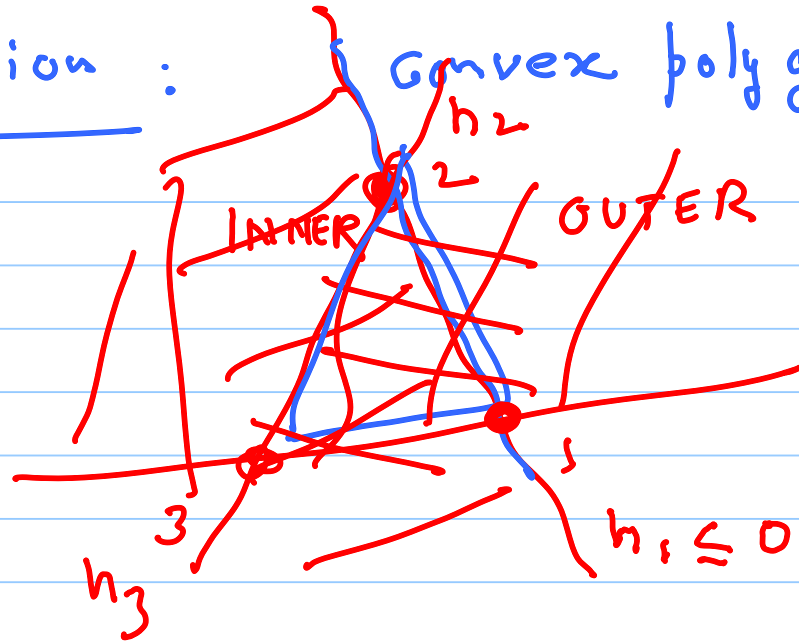


Digression : Convex polygon

1) implicit

2) explicit





$h_1 \le 0$

$h_2 \le 0$

$h_3 \le 0$

Boundary Reps.

eg. rep: list vertices CCW

vertex 1	vert 2	vert 3
x_1, y_1	x_2, y_2	x_3, y_3

$O(n)$

$$CB_\theta = \{ \underset{\in R^2\,(x,y)}{q} : A(q) \cap B \neq \phi \}$$

Cspace is R^2

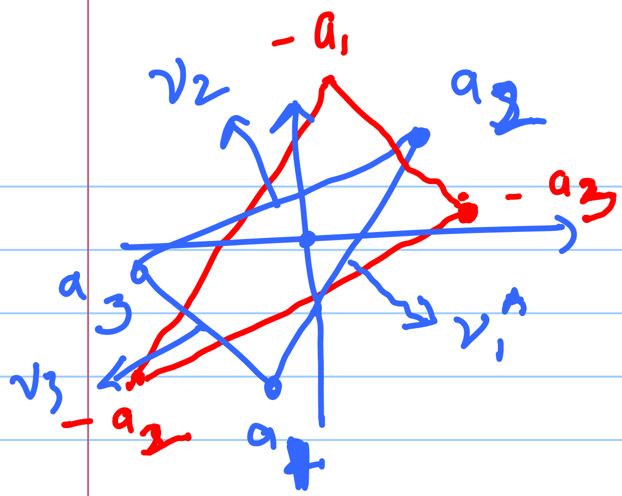
$$A_\theta$$

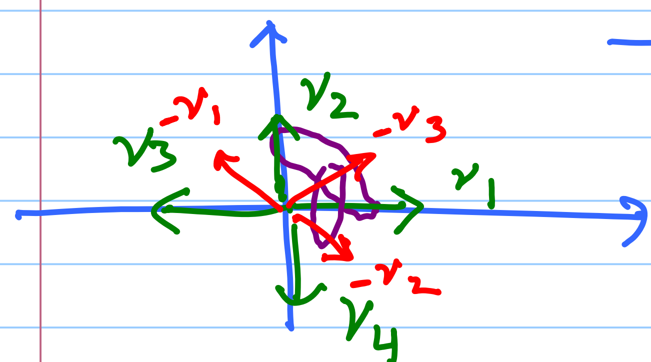
$$= \boxed{B \ominus A_\theta}$$

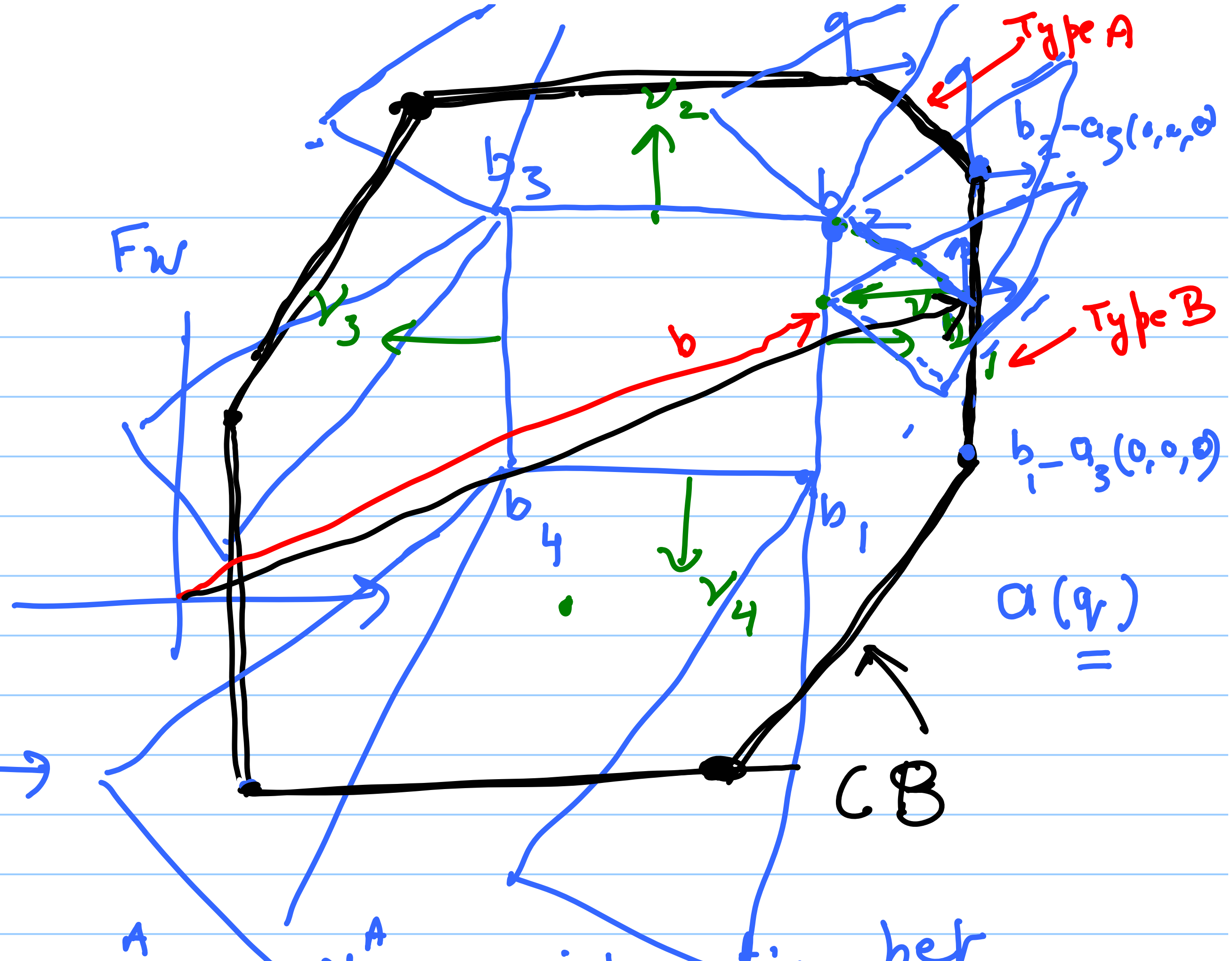
Alg for comp $B - A_\theta$:

A, B convex

$$\boxed{\Pi(A \oplus B) = \Pi(A) \oplus \Pi(B)}$$

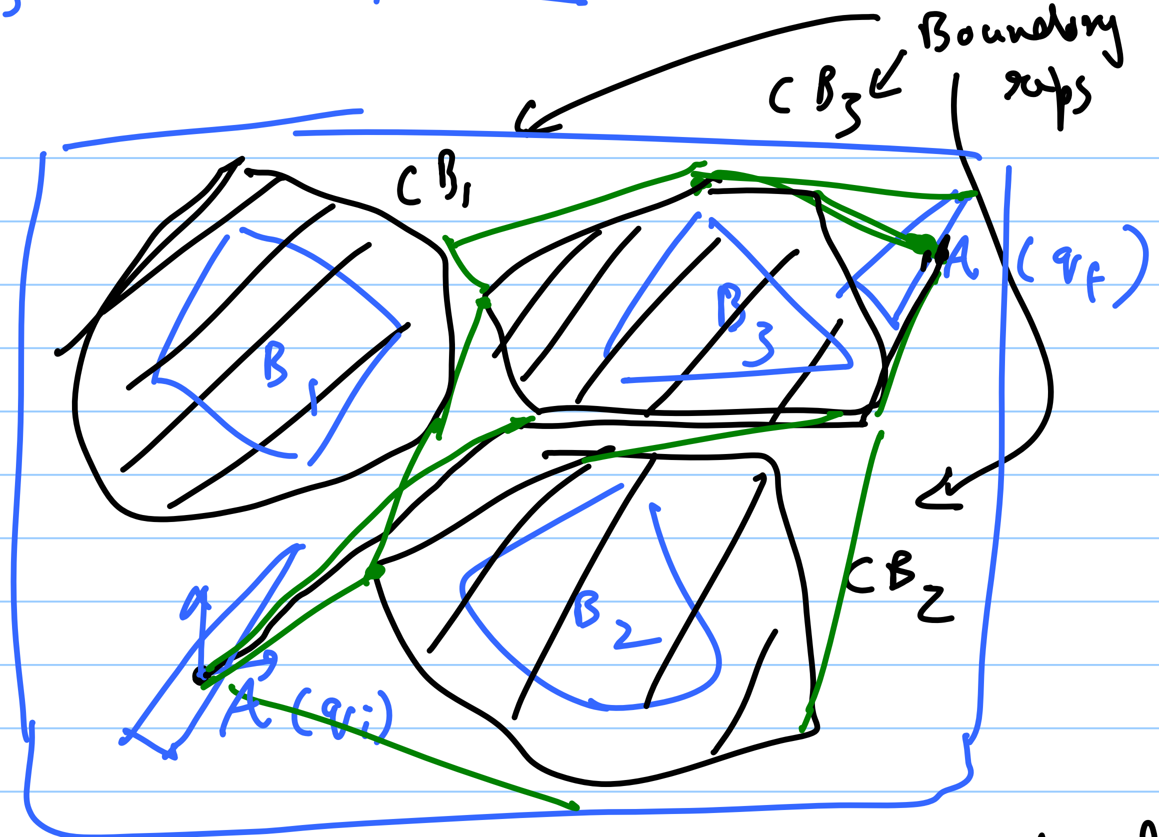






1) $\underbrace{\nu_1^B}_{\text{edge } b_1 b_2}$ is bet $\underbrace{-\nu_2^A + -\nu_3^A}_{\text{vert } a_3}$ $\Rightarrow$ intersection bet edge $b_1 b_2$ + vertix a_3

2) – v_3^{B} is bet v_1^{B} & v_2^{B}



Use visibility graph among polygonal C_{obs} to get shortest path.

~~Alg.~~ Completeness types for path plng

① Complete: if a soln. (coll. free) path exists, the alg. finds it and returns no soln. otherwise.

2. **Resolution complete**: if a soln exists with a $\boxed{\text{certain resolution}}^{\varepsilon}$ it will find it, and return no otherwise.

(choset) (Lavalle's book)

↳ no ε notion.

but alg. may not terminate for no soln.

3. **probabilistic complete**: "sampling"

will find a soln with Prob → 1 if it exists.

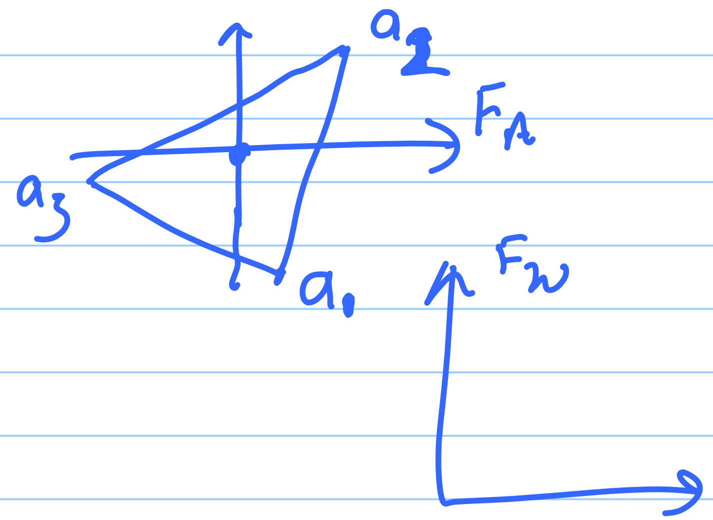
will not terminate for no soln.!

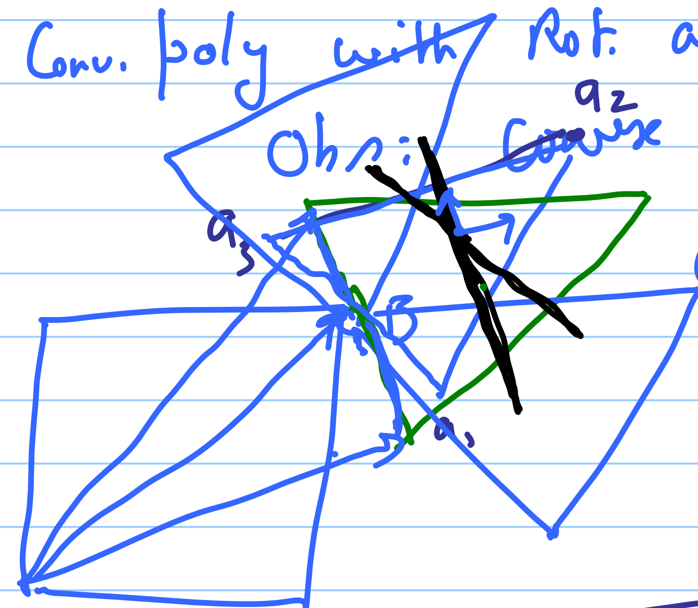
More gen. Case : Conv. poly with Rot. allowed: A

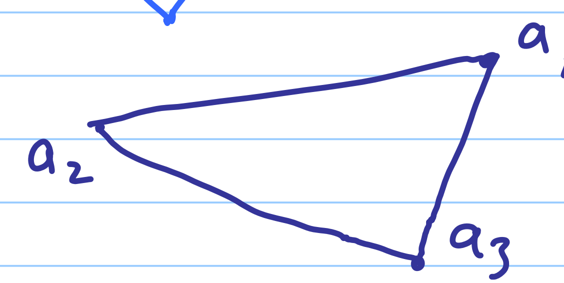
$q : (x, y, \theta)$

Obs: Convex polys

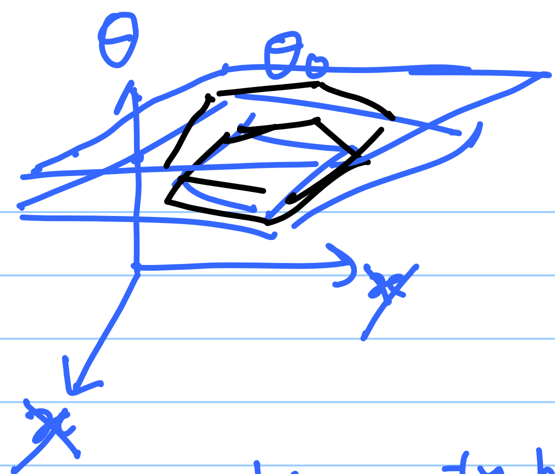
$C\mathcal{B}$??

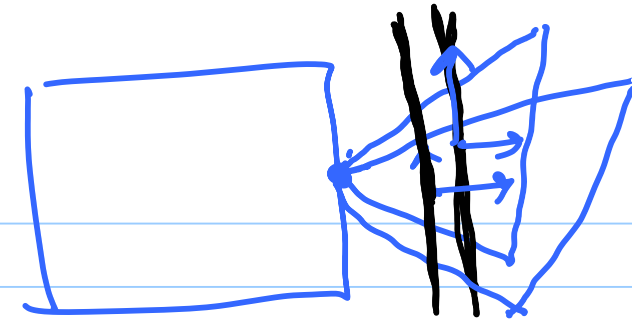






1) we know for fixed θ : $C\mathcal{B}[A_\theta]$ is a convex poly





two types of interaction:

1) edge of A, vertex of B $\longrightarrow$ type A ✓ "rotation of edge of A"

2) vert. of A, edge of B $\longrightarrow$ type B (around vertex B) $\rightarrow$ trans. par. to edge of B

What is the overall description rep. of CB ??

Diff. way:

1) Explicit Boundary rep: Avnaim + Boissonat

eqns valid
How?

give a q value

2) predicate type. ✓

$q \in CB$
or
$q \notin CB$

1) eqn. describing, say a type A int.

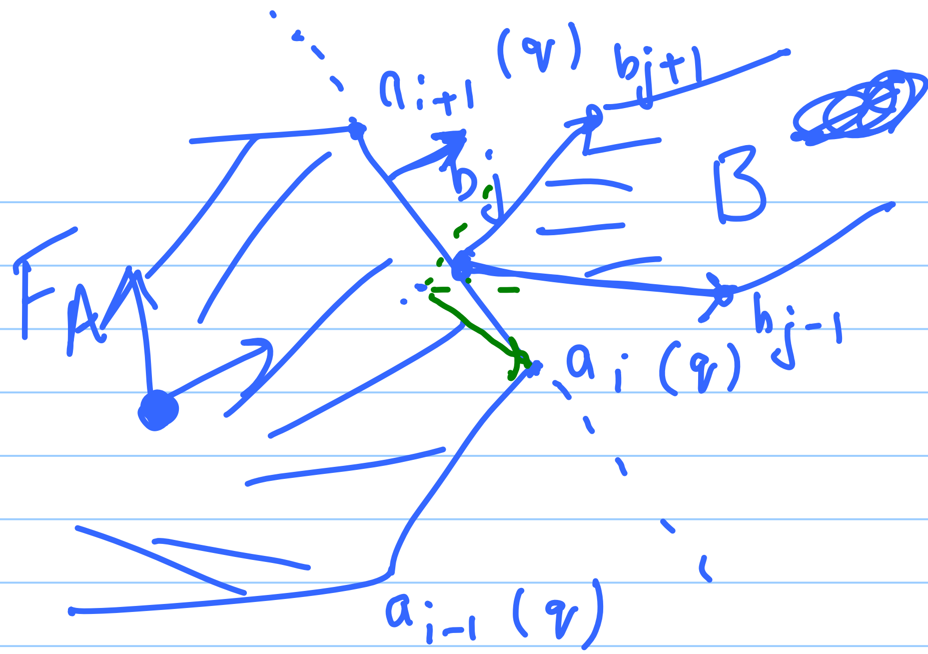
1) valid over a θ domain: interval
same eqn type A, type B

2) x, y : finite edges

edge i (A)
vert j (B)

$\boxed{f^{A}_{ij}(x, y, \theta) = 0}$

$\left.\begin{array}{l} [\theta_1, \theta_2] \\ (x, y, range) \end{array}\right\}$



$\text{Appl}_{ij}(q)$: $\begin{cases} \nu_i^A \cdot (b_{j-1} - b_j) \geq 0 \\ \text{and} \\ \nu_i^A \cdot (b_{j+1} - b_j) \geq 0 \end{cases}$

$\text{Appl}_{ij}(q)$ orient. indep. of transl. by θ

patch surface eqn: b_j lies on edge $a_i a_{i+1}$

$$f_{ij}^A(q) = \nu_i^A(q) \cdot [b_j - a_i(q)] = 0$$

$\leq$ for inside C_{obs}

$$\boxed{\mathrm{CONST}^A_{ij}(q) : \quad \mathrm{Appl.}_{ij}(q) \Longrightarrow f^A_{ij}(q) \leq 0}$$

write it for all i, j pairs

given q: can easily test if $\mathrm{CONST}^A_{ij}(q)$ is true

hence can easily determine if $q \in C_{obs}$

$\mathrm{CONST}^B_{ij}(q)$:

$$CB = \{ q \in C : A(q) \cap B \neq \phi \}$$

$$CB(q) = \underbrace{\left(\bigwedge_{i,j} \mathrm{CONST}^{A}_{i,j}(q) \right) \wedge \left(\bigwedge_{i,j} \mathrm{CONST}^{B}_{i,j}(q) \right)}$$

|| for convex robots
convex obs

m vertices in robot
n " " obs.

<u>Complexity</u> of clar $CB(q)$: $2mn$
or $O(mn)$

3-D Case:

face vertex
vertex face
edge edge

$(x, y, z, \theta, \phi, \psi)$

bottom line: C-obs are highly non-linear, complex entities, diff. to compute explicit description.

how do you now design
pthr plng algorithms?